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MATH 1470 Fall 2003 Tintera

TEST 3: Two-Species, Epidemic and Linear Programming Models. Covers Chapters 7-9

You may use calculators and one 8.5 by 11 inch page of notes. Please show all work on this test booklet. Partial credit is awarded only for work shown. Each problem is worth as indicated. Good luck!

1. An incomplete S-I-R model for a measles epidemic is as follows: $\frac{\Delta I}{\Delta t} = 0.0004SI - 0.2I$

a) (10 points) Write the change equations for S and R to complete the model

$$\frac{\Delta S}{\Delta t} = -0.0004\,SI$$

$$\frac{\Delta R}{\Delta t} = 0.2I$$

b) (5 points) According to this model, how long does it take to recover from the measles?

$b = 0.2 = \frac{1}{\text{recovery time}}$ So recovery time $= \frac{1}{.02} = \underline{\underline{5}}$

It takes 5 days to recover from the measles according to this mode.

c) (5 points) On one day there are 1000 Susceptible people and 20 Infected and none are Recovered. How many will there be the next day in each category.

$\Delta S = -.0004(1000)(20) = (-.4)(20) = -8$

So the next day $S = 1000 - 8 = 992$

$\Delta R = 0.2(20) = 4$. So the next day $R = 0 + 4 = 4$

$\Delta I = +8 + -4 = +4$. So the next day $I = 20 + 4 = 24$.

d) (5 points) For which values of S will I increase (ie, have $\Delta I > 0$) ?

For $\Delta I > 0$ $\quad 0.0004\,SI - 0.2\,I > 0$

or $I\,(0.0004\,S - 0.2) > 0$

Since $I > 0$ $\quad 0.0004\,S - 0.2 > 0$

$0.0004\,S > 0.2$

$S > \frac{0.2}{0.0004} = \frac{2000}{4} = 50$

So I will ~~with~~ increase any time $S > 500$

2. Consider the following two-species model:

$$\frac{\Delta x}{\Delta t} = 0.6x - 0.003x^2 - 0.006xy, \qquad \frac{\Delta y}{\Delta t} = 0.6y - .003y^2 - 0.001xy$$

a) (5 points) Is either species logistic? How can you tell?

Both are logistic since each has a negative logistic (x^2, or y^2) term.

b) (5 points) Is this a predator prey model or competing species model? How can you tell?

Since both species have negative interaction terms (xy) they both ~~loos~~ decrease because of the interaction. So the model is competing species.

e) (5 points) If the starting populations are x = 100 and y = 100, will the species x and y increase or decrease in the near future? For $x = 100$, $y = 100$

$$\frac{\Delta x}{\Delta t} = 0.6(100) - .003(100)^2 - 0.006(100)^2 = 60 - 30 - 60 = -30$$

$$\frac{\Delta y}{\Delta t} = 0.6(100) - .003(100)^2 - 0.001(100)^2 = 60 - 30 - 10 = +20$$

So x will decrease and y will increase

f) (10 points) Either

(i) Compare and contrast the two species in terms of their equations. How do they compare with regard to interactions? How do they compare with regard to other terms?

(ii) Or Sketch a phase plane for the two species.

(i) Both ~~two~~ species have identical coefficients for birth (0.6) and logistic ~~terms~~ (-0.003) terms. They differ in their ~~logistic~~ interaction terms. Species x has a more negative coefficient (-.006) than does species Y (-.001). So the species are similar biologically except for their interaction. Species y is a better competitor. We'd expect y to dominate

ii $\Delta x = 0 \Leftrightarrow x(.6 - .003x - .006y) = 0$

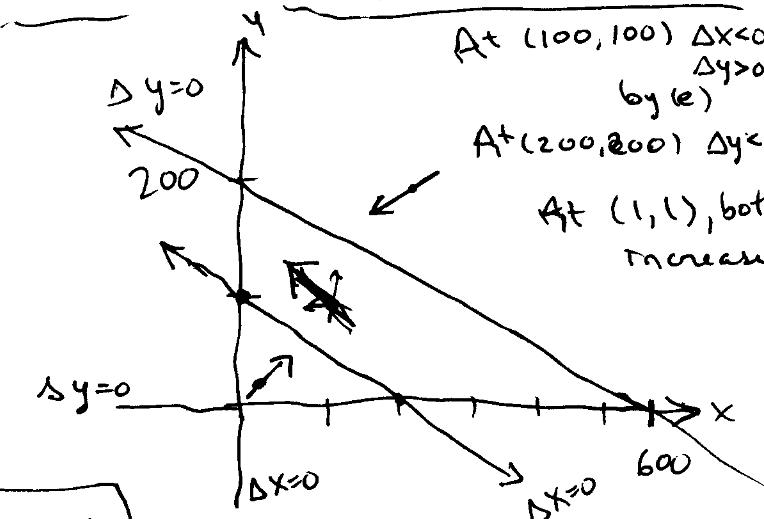
$x = 0$ or $.6 = .003x + .006y$

$(0, 100)$ $(200, 0)$

$\Delta y = 0 \Leftrightarrow y(.6 - .003y - .001x) = 0$

$y = 0$ or $.6 = .003y + .001x$

$(0, 200)$ $(600, 0)$



At (100,100) $\Delta x < 0$, $\Delta y > 0$ by (e)

At (200,200) $\Delta y < 0$

At (1,1), both increase

3. You and your roommate got a contract to advertise local sports franchises this summer. You will walk around with a sandwich board showing off arena football on the front and minor league baseball on the back. Each minute you spend downtown, you will be seen by 5 people interested in football and 12 people interested in baseball. Each minute spent at the beach will expose you to 10 people interested in football and 7 people interested in baseball. You will be seen by 1 person interested in football and 5 interested in baseball for each minute spent at the university. Your contract requires you to be exposed to 1500 people interested in football and 2000 people interested in baseball. Also, you need to spend more time at the university than at the beach. Of course, you want to spend part of your time this summer doing other things. Decide how to minimize the total time you spend wearing the sandwich board.

DO NOT SOLVE THIS PROBLEM. Instead name:

a) (6 points) Appropriate variables to solve the problem (eg, x1 = inches of rain, x2 = miles of flooded streets, x3 = dollars in flood damage).

d = time (in minutes) spent down town

b = ... at beach

u = ... university

b) (7 points) An objective function (with formula) and indicate whether it should be minimized or maximized.

Minimize time $= d + b + u$

c) (12 points) Constraints on the solutions to the objective function. You don't need to include non-negativity constraints.

Univ more than beach $\quad u \geq b$

Football min $\quad 5d + 10b + 1 \cdot u \geq 1500$

Baseball min $\quad 12d + \qquad \geq 2000$

4. (25 points) For the linear program

Maximize $Z = 6x + 7y$

subject to

$5x + 14y \le 70$

$10x + 12y \ge 60$

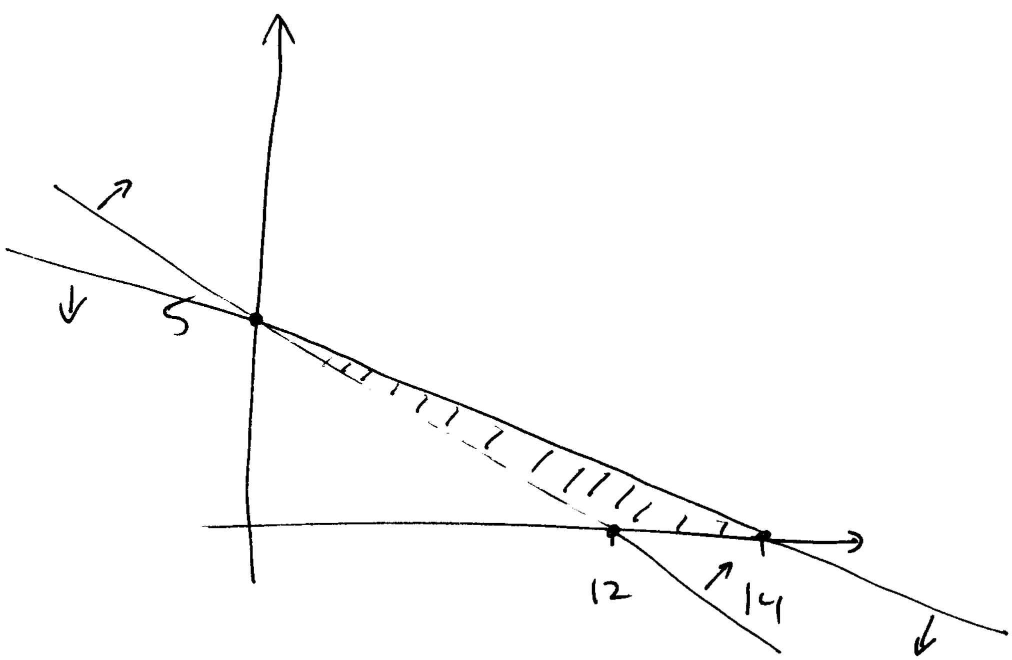
$x, y > 0$

solve the problem by graphing. Partial credit is awarded for work shown. Solution: Z = 84 is a maximum for x = 14 and y = 0

(1) Graph Constraints.

Below $5x + 14y = 70$ (0, 5); (14, 0)

Above $10x + 12y = 60$ (0, 5) (12, 0)



(2) Corner points (0, 5) (12, 0) (14, 0).

(3) Choose max Z.

x	y	$Z = 6x + 7y$
0	5	$6(0) + 7(5) = 35$
12	0	$6(12) + 7(0) = 72$
14	0	$6(14) + 7(0) = 84 \leftarrow$